HW. # 9

Homework problems are taken from several textbooks. The problems are color coded to indicate level of difficulty. The color green indicates an elementary problem, which you should be able to solve effortlessly. Yellow means that the problem is somewhat harder. Red indicates that the problem is hard. You should attempt the hard problems especially.

Calculate the first partial derivatives of each function with respect to the independent variables. If a point is given, evaluate the partials at that point.

1. $g(x, y) = x^2 y^3 - x^3 y^2$; (-2, 3)

2. $f(x, y) = (x - y)\cos(x + y)$; $(\frac{\pi}{2}, \frac{3\pi}{2})$

3. $f(x, y) = e^{xy} - ye^x$

4. $h(x, y) = \dfrac{1}{\sqrt{x^2 + y^2}}$

5. $f(x, y, z, w) = \ln\left(\dfrac{x + y}{z - w}\right)$; (5, 3, 2, -2)

6. $f(x_1, x_2, x_3, x_4, x_5) = x_1 \cos^{-1}(x_2 x_3) + \tan^{-1}\left(\dfrac{x_4}{x_5}\right)$

7. $M(x_1, x_2, ..., x_n) = \sum_{i=1}^{n} a_i x_i$

8. $G(x_1, x_2, ..., x_n) = \sqrt{\sum_{i=1}^{n} x_i^2}$

9. The ideal gas law states that for a gas confined to a container with volume V at temperature T, the pressure P exerted on the container wall is given by P = RT/V, where R is an empirically determined constant. (In this equation, volume is measured in cubic centimeters and temperature in Kelvin.) Suppose that an ideal gas is in a vessel with volume 1000 cubic centimeters at 600 K. Which will have a bigger effect on pressure, a small decrease in volume or a small increase in temperature?

10. For three resistors, with resistances r_1, r_2, and r_3, wired in parallel, the effective resistance R is given by $R = \dfrac{1}{\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}}$. Suppose that the resistances are 10, 15, and 20 ohms, respectively. To which resistor is the effective resistance most sensitive?

Calculate all of the second-order partials of the given functions.

11. $u(x, y) = 4x^3y^4 + x^2y^5 + xy$

12. $v(x, y) = \sin(x)\cos(y)$

13. $w(x, y) = e^{xy}$

14. $f(x, y) = x\tan^{-1}\left(\dfrac{x}{y}\right)$

Calculate the Jacobian matrix for the given function at the indicated point. Then write a formula for the total derivative.

15. $f(x, y) = 8x - 7y + 2$; **a** = (1, -3)

16. $g(x, y) = \dfrac{x - y}{x + y}$; **a** = (1/2, 3/2)

17. $p(x_1, x_2, x_3, x_4, x_5, x_6) = \dfrac{x_1^2 + x_2^2 + x_3^2}{x_4^2 + x_5^2 + x_6^2}$; **a** = (3, 4, 5, 1, 1, -1)

18. $k(x, y, z) = (z^2 - y^2, x^2 - z^2, y^2 - x^2)$; **a** = (1, 2, 3)

19. $g(x, y) = \left(\tan^{-1}\left(\dfrac{y}{x}\right), \dfrac{x}{x^2 + y^2}, \dfrac{y}{x^2 + y^2}\right)$; **a** = (1, 1)

20. v(t) = $(\sin t, \cos 2t, \sin 3t)$; a = $\dfrac{\pi}{6}$

Calculate *df*.

21. $f(x, y) = 2x^2 + xy + y^2 - 10$

22. $f(x,y) = \ln\sqrt{e^x - y}$

Use linear approximation to approximate a suitable function f(x, y) and thereby estimate the following:

23. $\left(0.99e^{0.02}\right)^8$

24. $\sqrt{(4.01)^2 + (3.98)^2 + (2.02)^2}$

25. A metal box with dimensions 1 meter by 1.5 meters by 2 meters is to be covered with gold leaf that is 0.001 mm thick. Approximately what volume of gold will be needed to do the job?

26. A mechanist has measured the height and radius of a cylindrical piece of metal to be 5.8 cm and 3.1 cm, respectively. If her measurements are in error by at most 0.002 and 0.001 cm, respectively, by approximately how much may the volume she calculates for the cylinder differ from its actual volume?

27. Suppose $f : U \subset R^n \to R^m$ is a linear transformation. What is the derivative of f? Justify your answer.

28. Suppose $f : U \subset R^n \to R^m$ is differentiable at the point **x** = **a**. Show that f is continuous at **x** = **a**. (Hint: $\lim_{x \to a} f(x) = f(a)$ if and only if $\lim_{x \to a} \|f(x) - f(a)\| = 0$. Let T be the derivative of f. Use triangle inequality to show that $\|f(x) - f(a)\| \le \|f(x) - f(a) - T(x-a)\| + \|T(x-a)\|$. Now what? Consult exercises 5 and 6 of HW#6)

29. Suppose $f : U \subset R^n \to R^m$ is differentiable at **x** = **a.** Show that the partial derivative $\frac{\partial f_i}{\partial x_j}$(**a**) exists for every $i \in \{1,\ldots,m\}$ and every $j \in \{1,\ldots,n\}$.

30. Let $f(x,y) = \begin{cases} xy(x^2 - y^2)/(x^2 + y^2) & (x,y) \ne (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

a) If (x, y) ≠ (0, 0), calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

b) Show that $\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 0$

c) Show that $\frac{\partial^2 f}{\partial x \partial y}(0,0) = 1$, $\frac{\partial^2 f}{\partial y \partial x}(0,0) = -1$

d) What went wrong? Why are the mixed partials not equal?